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# OSCILLATION OF SECOND-ORDER NONLINEAR NEUTRAL DYNAMIC EQUATIONS

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**Abstract:** The aim of this paper is to consider the oscillation of the second-order nonlinear neutral dynamic equation

$$\left(r(t)|y^{\Delta}(t)|^{\alpha-1}y^{\Delta}(t)\right)^{\Delta}+q(t)|x(t)|^{\beta-1}x(t)=0$$

on an arbitrary time scale $\mathbb{T}$, where $y(t):=x(t)+p(t)x(\tau(t))$ and $\alpha,\beta>0$ are constants. We obtain some oscillation criteria for the equation when $\beta>\alpha,\beta=\alpha$ and $\beta<\alpha$, respectively.

Our results improve and extend some known results where $p(t)\equiv 0$ and $\alpha,\beta$ are quotients of odd positive integers.

**AMS Subject Classification:** 34N05, 39A10
**Key Words:** oscillation, second-order nonlinear neutral dynamic equation, time scale

## 1. Introduction

The study of dynamic equations on time scales is a fairly new subject, and research in this area is rapidly growing. It has been created in order to unify continuous and discrete analysis, and it allows a simultaneous treatment of differential and difference equations. Dynamic equations on time scales have an enormous potential for applications in biology, engineering, economics, physics, neural networks, social sciences and so on. For instance, it can model insect

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populations that are continuous while in season, die out in say winter, while their eggs are incubating or dormant, and then hatch in a new season, giving rise to a nonoverlapping population (see [9]).

Recently, there has been increasing interest in obtaining sufficient conditions for the oscillation and nonoscillation of solutions of different classes of dynamic equations on time scales, and we refer the reader to the papers [1, 10, 8, 11, 3, 4, 5, 6, 7, 13, 2, 12] and the references cited therein. Thereinto, Saker [8] established some oscillation criteria for the second-order half-linear dynamic equation

$$\left(r(t)\left(x^{\Delta}(t)\right)^{\alpha}\right)^{\Delta}+q(t)x^{\alpha}(t)=0 \tag{1}$$

on time scales, where $\alpha>1$ is an odd positive integer, and $r$ and $q$ are positive rd-continuous functions.

Hassan [11] considered the same Equation (1), where $\alpha$ is a quotient of odd positive integers, and obtained some sufficient conditions for the oscillation. Hassan [11] improved and extended the results of Saker [8].

Grace et al. [12] studied the oscillation of the second-order nonlinear dynamic equation

$$\left(r(t)\left(x^{\Delta}(t)\right)^{\alpha}\right)^{\Delta}+q(t)x^{\beta}(t)=0 \tag{2}$$

on time scales, where $\alpha,\beta$ are quotients of odd positive integers, and $r$ and $q$ are positive rd-continuous functions. Grace et al. [12] gave some new oscillation results for (2) when $\beta>\alpha, \beta=\alpha$ and $\beta<\alpha$, respectively.

Following the above-mentioned research trend, in this paper we investigate the oscillation of the nonlinear second-order neutral dynamic equation

$$\left(r(t)\left|\left[x(t)+p(t)x(\tau(t))\right]^{\Delta}\right|^{\alpha-1}\left[x(t)+p(t)x(\tau(t))\right]^{\Delta}\right)^{\Delta}+q(t)|x(t)|^{\beta-1}x(t)=0 \tag{3}$$

on an arbitrary time scale $\mathbb{T}$, where $\alpha,\beta>0$ are constants, $r$ and $q$ are positive rd-continuous functions on time scale interval $[t_0,\infty)$, $0\le p(t)\le 1$ for $t\in[t_0,\infty)$, $\tau:\mathbb{T}\to\mathbb{T}$ satisfies $\tau(t)\le t$ for $t\in\mathbb{T}$ and $\lim_{t\to\infty}\tau(t)=\infty$. Since the oscillatory behavior of solutions near infinity is our primary concern, we make the assumption that $\sup\mathbb{T}=\infty$.

Obviously, (1) and (2) are special cases of (3), and all the results of Saker [8], Hassan [11] and Grace et al. [12] can not be applied to (3) when $p(t)\not\equiv 0$ or $\alpha,\beta$ are not equal to quotients of odd positive integers. Therefore, it is of great interest to study (3) when $0\le p(t)\le 1$ and $\alpha,\beta>0$ are constants. The purpose of this paper is to establish some new oscillation criteria for (3). Our results extend and improve the results of Saker [8], Hassan [11] and Grace et al. [12].

Recall that a solution of (3) is a nontrivial real function $x$ such that $x(t)+p(t)x(\tau(t)) \in C^1_{rd}[t_x,\infty)$ and $r(t)\Big|\big[x(t)+p(t)x(\tau(t))\big]^{\Delta}\Big|^{\alpha-1}\big[x(t)+p(t)x(\tau(t))\big]^{\Delta} \in C^1_{rd}[t_x,\infty)$ for a certain $t_x \ge t_0$ and satisfying (3) for $t \ge t_x$. Our attention is restricted to those solutions of (3) which exist on half-line $[t_x,\infty)$ and satisfy $\sup\{|x(t)| : t > t_*\} > 0$ for any $t_* \ge t_x$. A solution $x$ of (3) is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is nonoscillatory. Equation (3) is said to be oscillatory if all its solutions are oscillatory.

In the sequel, for convenience, when we write a functional inequality without specifying its domain of validity we assume that it holds for all sufficiently large $t$.

## 2. Main Results

**Theorem 2.1.** *Let $\beta > \alpha$, and suppose the following conditions hold:*

$$\int_{t_0}^{\infty} \left(\frac{1}{r(t)}\right)^{1/\alpha} \Delta t = \infty, \tag{1}$$

$$0 < Q(t) := \int_t^{\infty} q(s)[1-p(s)]^{\beta}\Delta s < \infty \quad for \quad t \in [t_0,\infty) \tag{2}$$

*and*

$$\int_{t_0}^{\infty} r^{-\frac{1}{\alpha}}(s) H^{\sigma}(s,c)\Delta s = \infty, \tag{3}$$

*where $c$ is an arbitrary positive constant and*

$$H(t,c) := \Big[Q(t) + c\int_t^{\infty} r^{-\frac{1}{\alpha}}(s)\big(Q^{\sigma}(s)\big)^{\frac{1+\alpha}{\alpha}}\Delta s\Big]^{1/\alpha}.$$

*Then (3) is oscillatory.*

*Proof.* Suppose that $x$ is a nonoscillatory solution of (3). Without loss of generality, we may assume that $x$ is an eventually positive solution of (3). Let

$$y(t) := x(t) + p(t)x(\tau(t)), \quad t \in \mathbb{T}. \tag{4}$$

Then there exists $t_1 \in [t_0,\infty)$ such that

$$y(t) \ge x(t) > 0, \quad t \in [t_1,\infty). \tag{5}$$

It follows from (3), (4) and (5) that

$$(r(t)|y^{\Delta}(t)|^{\alpha-1}y^{\Delta}(t))^{\Delta} = -q(t)x^{\beta}(t) < 0, \quad t \in [t_1, \infty). \tag{6}$$

Thus, $r(t)|y^{\Delta}(t)|^{\alpha-1}y^{\Delta}(t)$ is strictly decreasing on $[t_1, \infty)$ and is eventually of same sign. We claim

$$y^{\Delta}(t) > 0, \quad t \in [t_1, \infty). \tag{7}$$

Assume on the contrary, then there exists $t_2 \in [t_1, \infty)$ such that $y^{\Delta}(t_2) \leq 0$. Take $t_3 > t_2$. Since $r(t)|y^{\Delta}(t)|^{\alpha-1}y^{\Delta}(t)$ is strictly decreasing on $[t_1, \infty)$, we have

$$\begin{aligned} r(t)|y^{\Delta}(t)|^{\alpha-1}y^{\Delta}(t) &\leq r(t_3)|y^{\Delta}(t_3)|^{\alpha-1}y^{\Delta}(t_3) \\ &:= M < r(t_2)|y^{\Delta}(t_2)|^{\alpha-1}y^{\Delta}(t_2) \leq 0, \quad t \in [t_3, \infty). \end{aligned}$$

Thus, we obtain $y^{\Delta}(t) \leq -(-M)^{\frac{1}{\alpha}}\big(\frac{1}{r(t)}\big)^{1/\alpha}$ for $t \in [t_3, \infty)$. Integrating both sides of the last inequality from $t_3$ to $t$, we get

$$y(t) - y(t_3) \leq -(-M)^{\frac{1}{\alpha}} \int_{t_3}^{t} \big(\frac{1}{r(s)}\big)^{1/\alpha} \Delta s, \quad t \in [t_3, \infty).$$

Letting $t \to \infty$ and noticing (1), we see that $\lim_{t\to\infty} y(t) = -\infty$. This contradicts (5). Hence, (7) holds. From (4) and (5), we conclude

$$x(t) = y(t) - p(t)x(\tau(t)) \geq y(t) - p(t)y(\tau(t)). \tag{8}$$

Since $\tau(t) \leq t$ and $y^{\Delta}(t) > 0$, we have $y(\tau(t)) \leq y(t)$. Therefore, from (8) we obtain

$$x(t) \geq y(t) - p(t)y(t) = [1 - p(t)]y(t). \tag{9}$$

From (6), (7) and (9), there exists $t_4 \in [t_3, \infty)$ such that

$$[r(t)(y^{\Delta}(t))^{\alpha}]^{\Delta} \leq -q(t)[1 - p(t)]^{\beta}y^{\beta}(t) \leq 0, \quad t \in [t_4, \infty). \tag{10}$$

Define the function $w$ by

$$w(t) = \frac{r(t)(y^{\Delta}(t))^{\alpha}}{y^{\beta}(t)}, \quad t \in [t_4, \infty). \tag{11}$$

It is easy to see that $w(t) > 0$ for $t \in [t_4, \infty)$. By the product and quotient rules for the delta derivative and then from (10) and (11), we get

$$\begin{aligned} w^\Delta &= (r(y^\Delta)^\alpha)^\Delta \frac{1}{y^\beta} + (r(y^\Delta)^\alpha)^\sigma (\frac{1}{y^\beta})^\Delta \\ &\le -q(1-p)^\beta - (r(y^\Delta)^\alpha)^\sigma \frac{(y^\beta)^\Delta}{y^\beta (y^\beta)^\sigma} \\ &= -q(1-p)^\beta - w^\sigma \frac{(y^\beta)^\Delta}{y^\beta} \quad \text{on } [t_4, \infty). \end{aligned} \tag{12}$$

By the Pötzsche chain rule (Bohner and Peterson [9], p. 32, Theorem 1.87), for $t \in [t_4, \infty)$ we obtain

$$\begin{aligned} (y^\beta(t))^\Delta &= \beta \Big\{ \int_0^1 [y(t) + h\mu(t) y^\Delta(t)]^{\beta-1} dh \Big\} y^\Delta(t) \\ &= \beta \Big\{ \int_0^1 [(1-h)y(t) + hy^\sigma(t)]^{\beta-1} dh \Big\} y^\Delta(t) \\ &\ge \begin{cases} \beta(y(t))^{\beta-1} y^\Delta(t), & \beta > 1, \\ \beta(y^\sigma(t))^{\beta-1} y^\Delta(t), & 0 < \beta \le 1. \end{cases} \end{aligned}$$

Thus, on $[t_4, \infty)$ we have

$$\frac{(y^\beta)^\Delta}{y^\beta} \ge \begin{cases} \beta \frac{y^\Delta}{y}, & \beta > 1, \\ \beta \frac{(y^\sigma)^{\beta-1}}{y^\beta} y^\Delta, & 0 < \beta \le 1. \end{cases} \tag{13}$$

Noticing the fact that $y$ is an increasing function on $[t_4, \infty)$ and $t \le \sigma(t)$, we get $y(t) \le y^\sigma(t)$ for $t \in [t_4, \infty)$. Therefore, it follows from (13) that

$$\frac{(y^\beta)^\Delta}{y^\beta} \ge \beta \frac{y^\Delta}{y^\sigma} \quad \text{on } [t_4, \infty) \text{ for } \beta > 0. \tag{14}$$

Using (14) in (12), we obtain

$$w^\Delta \le -q(1-p)^\beta - \beta w^\sigma \frac{y^\Delta}{y^\sigma} \quad \text{on } [t_4, \infty). \tag{15}$$

Since $r^{\frac{1}{\alpha}} y^\Delta$ is a decreasing function on $[t_4, \infty)$ and $t \le \sigma(t)$, we conclude $r^{\frac{1}{\alpha}} y^\Delta \ge (r^{\frac{1}{\alpha}} y^\Delta)^\sigma$ on $[t_4, \infty)$. Hence, from (11) we obtain

$$y^\Delta \ge r^{-\frac{1}{\alpha}} (w^\sigma)^{\frac{1}{\alpha}} (y^\sigma)^{\frac{\beta}{\alpha}} \quad \text{on } [t_4, \infty). \tag{16}$$

Substituting (16) in (15), we have

$$w^{\Delta} \le -q(1-p)^{\beta} - \beta r^{-\frac{1}{\alpha}}(w^{\sigma})^{1+\frac{1}{\alpha}}(y^{\sigma})^{\frac{\beta}{\alpha}-1} \quad \text{on } [t_4,\infty).$$

Integrating both sides of the last inequality from $t$ to $u$ ($u \ge t \ge t_4$) and letting $u \to \infty$, we obtain

$$w(t) \ge \int_t^{\infty} q(s)[1-p(s)]^{\beta}\Delta s + \beta\int_t^{\infty} r^{-\frac{1}{\alpha}}(s)(w^{\sigma}(s))^{1+\frac{1}{\alpha}}(y^{\sigma}(s))^{\frac{\beta}{\alpha}-1}\Delta s, \quad t \in [t_4,\infty).$$

It is clear that $w(t) \ge Q(t) := \int_t^{\infty} q(s)[1-p(s)]^{\beta}\Delta s$ for $t \ge t_4$. Thus, we get

$$w(t) \ge Q(t) + \beta\int_t^{\infty} r^{-\frac{1}{\alpha}}(s)(Q^{\sigma}(s))^{1+\frac{1}{\alpha}}(y^{\sigma}(s))^{\frac{\beta}{\alpha}-1}\Delta s, \quad t \in [t_4,\infty). \tag{17}$$

Since $\beta > \alpha$ and $y$ is an increasing function on $[t_4,\infty)$, there exist a $t_5 \ge t_4$ and a positive constant $c_1$ such that

$$(y^{\sigma}(s))^{\frac{\beta}{\alpha}-1} \ge c_1, \quad s \in [t_5,\infty). \tag{18}$$

Using (18) in (17), we see

$$w(t) \ge Q(t) + \beta c_1\int_t^{\infty} r^{-\frac{1}{\alpha}}(s)(Q^{\sigma}(s))^{1+\frac{1}{\alpha}}\Delta s := H^{\alpha}(t,c), \quad t \in [t_5,\infty),$$

where $c := \beta c_1$. Since $r(y^{\Delta})^{\alpha}$ is decreasing on $[t_5,\infty)$ and $t \le \sigma(t)$, we have $r(y^{\Delta})^{\alpha} \ge (r(y^{\Delta})^{\alpha})^{\sigma}$ on $[t_5,\infty)$. Therefore, we obtain

$$\frac{r(y^{\Delta})^{\alpha}}{(y^{\sigma})^{\beta}} \ge \frac{(r(y^{\Delta})^{\alpha})^{\sigma}}{(y^{\sigma})^{\beta}} = w^{\sigma} \ge (H^{\alpha}(t,c))^{\sigma} \quad \text{on } [t_5,\infty),$$

which implies

$$(y^{\sigma})^{-\delta}y^{\Delta} \ge r^{-\frac{1}{\alpha}}H^{\sigma}(t,c) \quad \text{on } [t_5,\infty), \tag{19}$$

where $\delta := \beta/\alpha > 1$. Applying the Pötzsche chain rule (Bohner and Peterson [9], p. 32, Theorem 1.87), we get

$$\begin{aligned}(y^{1-\delta})^{\Delta}(t) &= (1-\delta)\left\{\int_0^1 [y(t)+h\mu(t)y^{\Delta}(t)]^{-\delta}dh\right\}y^{\Delta}(t)\\ &= (1-\delta)\left\{\int_0^1 [(1-h)y(t)+hy^{\sigma}(t)]^{-\delta}dh\right\}y^{\Delta}(t)\\ &\le (1-\delta)(y^{\sigma}(t))^{-\delta}y^{\Delta}(t) \quad \text{on } [t_5,\infty).\end{aligned}$$

Thus, we obtain

$$\frac{(y^{1-\delta})^{\Delta}}{1-\delta} \geq (y^{\sigma})^{-\delta} y^{\Delta} \quad \text{on } [t_5, \infty). \tag{20}$$

From (19) and (20), we conclude

$$(y^{1-\delta})^{\Delta}/(1-\delta) \geq r^{-\frac{1}{\alpha}} H^{\sigma}(t,c) \quad \text{on } [t_5, \infty).$$

Integrating both sides of the last inequality from $t_5$ to $t$ ($t \geq t_5$), we have

$$\int_{t_5}^{t} r^{-\frac{1}{\alpha}}(s) H^{\sigma}(s,c) \Delta s \leq \frac{(y^{1-\delta})(t_5)}{\delta - 1}.$$

Letting $t \to \infty$, we see $\int_{t_5}^{\infty} r^{-\frac{1}{\alpha}}(s) H^{\sigma}(s,c) \Delta s \leq \frac{(y^{1-\delta})(t_5)}{\delta-1} < \infty$, which contradicts (3). Hence, the proof is complete. □

**Theorem 2.2.** *Let $\beta = \alpha$, and suppose that (1) and (2) hold. If*

$$\limsup_{t \to \infty} \left( \int_{t_0}^{t} r^{-\frac{1}{\alpha}}(s) \Delta s \right) H(t, \alpha) > 1, \tag{21}$$

*where $H$ is defined as in Theorem 2.1, then (3) is oscillatory.*

*Proof.* Assume that $x$ is a nonoscillatory solution of (3). Without loss of generality, we may assume that $x$ is an eventually positive solution of (3). Proceeding as in the proof of Theorem 2.1, we find that (17) takes the form

$$\begin{aligned} w(t) &\geq Q(t) + \alpha \int_{t}^{\infty} r^{-\frac{1}{\alpha}}(s) (Q^{\sigma}(s))^{1+\frac{1}{\alpha}} \Delta s \\ &:= H^{\alpha}(t, \alpha), \quad t \in [t_4, \infty). \end{aligned} \tag{22}$$

Since $r^{\frac{1}{\alpha}} y^{\Delta}$ is decreasing on $[t_4, \infty)$, we have

$$\begin{aligned} y(t) = y(t_4) + \int_{t_4}^{t} y^{\Delta}(s) \Delta s &= y(t_4) + \int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s) \Big( r^{\frac{1}{\alpha}}(s) y^{\Delta}(s) \Big) \Delta s \\ &\geq r^{\frac{1}{\alpha}}(t) y^{\Delta}(t) \int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s) \Delta s, \quad t \in [t_4, \infty). \end{aligned}$$

Therefore, we obtain

$$\frac{r^{\frac{1}{\alpha}}(t) y^{\Delta}(t)}{y(t)} \leq \left( \int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s) \Delta s \right)^{-1}, \quad t \in [t_4, \infty). \tag{23}$$

From (22), (11) and (23), we get

$$H(t,\alpha) \le w^{\frac{1}{\alpha}}(t) = \frac{r^{\frac{1}{\alpha}}(t)y^{\Delta}(t)}{y(t)} \le \left(\int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s\right)^{-1}, \quad t \in [t_4,\infty).$$

Thus, we find

$$\left(\int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s\right)H(t,\alpha) \le 1, \quad t \in [t_4,\infty).$$

Taking lim sup of both sides of the last inequality as $t \to \infty$, we get a contradiction to (21). The proof is complete. $\square$

**Theorem 2.3.** *Let $\beta < \alpha$, and suppose that (1) and (2) hold. If*

$$\limsup_{t\to\infty} Q^{(\alpha-\beta)/(\alpha\beta)}(t)\left(\int_{t_0}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s\right)\left[Q(t)+c\int_{t}^{\infty} r^{-\frac{1}{\alpha}}(s)(Q^{\sigma}(s))^{1+\frac{1}{\beta}}\Delta s\right]^{1/\alpha} = \infty \quad (24)$$

*for every constant $c > 0$, then (3) is oscillatory.*

*Proof.* Suppose that $x$ is a nonoscillatory solution of (3). Without loss of generality, we may assume that $x$ is an eventually positive solution of (3). Proceeding as in the proof of Theorem 2.1 to obtain (5)–(7) and (17). It follows from (17) that $w(t) \ge Q(t)$ on $[t_4,\infty)$. Therefore, from (11) we have $r^{1/\alpha}y^{\Delta} \ge y^{\beta/\alpha}Q^{1/\alpha}$ on $[t_4,\infty)$. Since $r^{1/\alpha}y^{\Delta}$ is a decreasing function on $[t_4,\infty)$, there exist $k > 0$ and $t_5 \ge t_4$ such that $k \ge r^{1/\alpha}y^{\Delta} \ge y^{\beta/\alpha}Q^{1/\alpha}$ on $[t_5,\infty)$. Thus, we get

$$y \le k^{\alpha/\beta}Q^{-1/\beta} \quad \text{on } [t_5,\infty). \quad (25)$$

Hence, we obtain

$$(y^{\sigma})^{(\beta-\alpha)/\alpha} \ge k^{(\beta-\alpha)/\beta}(Q^{\sigma})^{(\alpha-\beta)/(\alpha\beta)} \quad \text{on } [t_5,\infty). \quad (26)$$

Using (26) in (17) and noticing the definition of $w$, we see

$$(y(t))^{(\alpha-\beta)/\alpha}\frac{r^{1/\alpha}(t)y^{\Delta}(t)}{y(t)} \ge \left[Q(t)+c\int_{t}^{\infty} r^{-\frac{1}{\alpha}}(s)(Q^{\sigma}(s))^{1+\frac{1}{\beta}}\Delta s\right]^{1/\alpha}, \quad t \in [t_5,\infty), \quad (27)$$

where $c := \beta k^{(\beta-\alpha)/\beta}$. Using (23) and (25) in (27), we find

$$k^{(\alpha-\beta)/\beta} Q^{(\beta-\alpha)/(\alpha\beta)}(t)\left(\int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s\right)^{-1} \geq \left[Q(t) + c\int_{t}^{\infty} r^{-\frac{1}{\alpha}}(s)(Q^{\sigma}(s))^{1+\frac{1}{\beta}}\Delta s\right]^{1/\alpha}$$

for $t \in [t_5, \infty)$. Therefore, we have

$$k^{(\alpha-\beta)/\beta} \geq Q^{(\alpha-\beta)/(\alpha\beta)}(t)\left(\int_{t_4}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s\right)\left[Q(t) + c\int_{t}^{\infty} r^{-\frac{1}{\alpha}}(s)(Q^{\sigma}(s))^{1+\frac{1}{\beta}}\Delta s\right]^{1/\alpha}$$

for $t \in [t_5, \infty)$. Taking lim sup of both sides of the last inequality as $t \to \infty$, we get a contradiction to (24). The proof is complete. □

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